Final Review

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Outline

Introduction

1. Introduction to Financial Economics(2 pricing theories **[HW1,Q1]**
2. Interest & Bond:
   1. IRR, NPV, yield **[HW1,Q2]**
   2. Spot Rate(pricing), Forward Rate **[HW1,Q3]**
   3. Duration
3. Stocks:
   1. DDM model**[HW2,Q1]**
   2. Dividend decision & Fisher Separation Theorem**[HW2,Q2Q3]**

Outline

CAPM

1. Preference: Mean-Variance Analysis**[HW3,Q1]**
2. Behavior: Market Portfolio & Two-fund separation**[HW3,Q2]**
3. Equilibrium: Partial, SML vs. CML
    1. Proof1: Quadratic Utility Function**[HW4,Q1]**
    2. Proof2: Portfolio Construction & Sharp Ratio**[HW4,Q2]**
4. Properties: CAPM
    1. Systematic vs. Idiosyncratic
    2. $E(r_i) - r_f = \beta_i(E(r_M) - r_f)$**[HW4,Q3]**
    3. Applications & Deficiency**[HW4,Q4]**

Outline

- Preference: Expected Utility(Lecture 8)
  - Expected Utility Theorem(Without Proof)**[HW5,Q1]**
  - Risk Aversion: ARA, RRA and common utility functions**[HW5,Q2Q3]**
- Behavior: Behavior under risks(Lecture 9)
  - Risk Assets [Different State]
    - Proposition1:$a^* > (=,<)0 \Leftrightarrow E\tilde{r} > (=,<)r_f$
    - Proposition2:$a^{*\prime}(w_0) > (=,<)0 \Leftrightarrow R_A'(\cdot) < (=,0)0(DARA, CARA, IARA)$
    - Proposition3:e$(w_0) = (>,<)1 \Leftrightarrow R_R'(\cdot) = (<,>)0(CRRA, DRRA, IRRA)$((Without Proof)
  - Risk and Savings [Different Time]
    - Determinacy case & Uncertainty case**[HW6,Q1]**
    - Proposition4:$s_A > (=,<)s_B \Leftrightarrow P_R(sR) < (=,>)2$
- Equilibrium: General Equilibrium(Lecture 10-11)
- Properties: C-CAPM(Lecture 12)

- Preference: Expected Utility(Lecture 8)
- Behavior: Behavior under risks(Lecture 9)
- Equilibrium: General Equilibrium(Lecture 10-11)
  - Asset market + Complete,Arrow-Debreu**[HW6,Q2Q3]**
  - Equilibrium in Complete Market**[HW6,Q4]**
  - Property of best risk sharing: Central Planner**[HW7,Q1Q2]**
    - Consumptions of all consumers are perfectly correlated
    - Consumptions only determined by aggregate risk
    - Idiosyncratic risk
  - Representative Consumer （HARA）
  - Asset prices in equilibrium
- Properties: C-CAPM(Lecture 12)
  - $E[\tilde{r}_j] = r_f + (E[\tilde{r}_j] - r_f)$
  - Risk-free rate: $r_f \approx \frac{1-\delta}{\delta} + R_R\bar{g} - \frac{1}{2}R_RP_R\sigma_g^2$
  - Risk premium:$E[\tilde{r}_j] - r_f = -\frac{\delta(1+r_f)}{u'(c_0)}cov(u'(\tilde{c_1}), \tilde{r}_j)$

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1. Introduction:Lect1-4
2. Absolute pricing->CAPM:Lect5-7
3. Absolute pricing->C-CAPM:Lect8-12
4. Relative pricing->Non-Arbitrage:Lect13-19
   - APT:(Lecture 13)
   - NA pricing principle:(Lecture 14-15)
   - Extension1:Multiperiod pricing(dynamic)(Lecture 16)
   - Extension2:Optimal stopping(Lecture 17)
   - Extension3:Continuous-time Finance(Lecture 18)
   - Application:Dynamic Hedging(Lecture 19)
5. Raitonal->Financial Frictions:Lect21-22
6. Irrational->Behavior Finance:Lect23
7. Financial Philosophy:Lect24-25

APT Model

- Assumption: $\tilde{r_i} = \bar{r_i} + \sum_{k=1}^{K} \beta_{i,k}\tilde{f_k} + \tilde{\epsilon}_i, \quad i = 1, 2, \cdots, N$

$$\tilde{r_p} = \sum_{i=1}^{N} w_i\tilde{r_i} = \sum_{i=1}^{N} w_i\bar{r_i} + (\sum_{i=1}^{N} w_i\beta_{i,1})\tilde{f_1} + \cdots + (\sum_{i=1}^{N} w_i\beta_{i,K})\tilde{f_K} + \sum_{i=1}^{N} w_i\tilde{\epsilon}_i$$

- Step1: Risk-free rate:

  $\tilde{r_{p0}} \approx \sum_{i=1}^{N} w_i(\beta_i)\bar{r_i} = r_f \Rightarrow \bar{r_i} = r_f + \sum_{k=1}^{K} \beta_{i,k}\lambda_k$

- Step2: Factor portfolio: $\tilde{r_{pk}} \approx \sum_{i=1}^{N} w_i'(\beta_i)\bar{r_i} + \tilde{f_k} \Rightarrow \lambda_k = \tilde{r_{pk}} - r_f - \tilde{f_k}$
- Conclusion:

  $\tilde{r_i} = r_f + \sum_{k=1}^{K} \beta_{i,k}((\bar{r_{pk}} - r_f) + \tilde{f_k}) + \tilde{\epsilon}_i, \quad i = 1, 2, \cdots, N$(Given $\bar{r_{pk}}, r_f$)**[HW8 Q1]**
- Remark: 因子组合 + 无风险利率 $\Rightarrow$ 其他资产定价

Outline

NA pricing principle

- Introduction: Future, Option and other derivatives **[HW8 Q2]**
  1. Forward price vs. Expectation of spot price in the future
  2. Put-call Parity
  3. Basic pricing idea: replicate bond/option, Risk Neutral World **[HW8 Q3]**

$$\Delta = \frac{C_u - C_d}{(u-d)S_0}, B = \frac{uC_d - dC_u}{e^r(u-d)}$$

- Fundamental Theorem of Asset Pricing

$$\text{N.A. complete} \Leftrightarrow \exists! \varphi \text{s.t.} P_j = \sum_{s=1}^{S} \varphi_s x_s^j = e^{-r} \sum_{s=1}^{S} \frac{\varphi_s}{\sum_{k=1}^{S} \varphi_k} x_s^j = e^{-r} E^Q[\tilde{x}^j]$$

[HW9 Q1]

Outline

Multiperiod pricing(Dynamic)

- Dynamic complete: Long-lived asset $\geq$ Maximum of successor node
- Law of iterated expectation
- Dynamic pricing:**[HW9 Q2、Q3]**
  1. Martingale: Define $\hat{S_t} = e^{-rt}S_t$ as deflated stock price, we have $E_0[\hat{S_2}] = E_0[\hat{S_1}] = \hat{S_0}$
  2. $q = \frac{e^r - d}{u - d}$
  3. $C_u = e^{-r}[qC_{uu} + (1-q)C_{ud}], C_d = e^{-r}[qC_{ud} + (1-q)C_{dd}]$
  4. $C_0 = e^{-2r}[q^2 C_{uu} + 2q(1-q)C_{ud} + (1-q)^2 C_d d]$

Outline

Bellman Equation(Dynamic)

- Problem1:

$$V(R,G) = max\{0, \frac{R}{R+G}[1+V(R-1,G)] + \frac{G}{G+R}[-1+V(R,G-1)]\}$$

[HW10 Q1]

- Problem2:

$$P_d = max\{max\{K - S_d, 0\}, \frac{1}{1+r}[qP_{ud} + (1-q)P_{dd}]\}$$

[HW10 Q2]

- Problem3:

$$V_s = min\{B_t, \frac{1}{1+r_s}[q(\bar{r}B_t + B_t - B_{t+1} + V_{su}) + (1-q)(\bar{r}B_t + B_t - B_{t+1} + V_{sd})]\}$$

[HW10 Q3]

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Dynamic Hedging

- Delta Hedge: $\Delta = \frac{\partial \Pi}{\partial S}$
- Greeks **[HW11 Q1]**
  1. Gamma: $\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 \Pi}{\partial S^2}$
  2. Vega: $\nu = \frac{\partial \Pi}{\partial \sigma}$
- Portfolio Insurance: replicate option**[HW11 Q2]**

$$\Delta = \frac{C_u - C_d}{(u-d)S_0}, B = \frac{uC_d - dC_u}{e^r(u-d)}$$

Consider cost???

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Acknowledge

60pt 计算 vs. 40pt 简答，请务必携带计算器

Thanks for your listening and sleeping!!!